Costly-Signaling Games: Rationality and Evolution (a classification)

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Workshop on Optimization, Game Theory, and Data Analysis (OGDA)

In celebration of Immanuel Bomze's 60th anniversary

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Some of Immanuel Bomze's work in evolutionary game theory:

- Lotka-Volterra equation and replicator dynamics: a two-dimensional classification. *Biological Cybernetics* (1983).
- Non-cooperative two-person games in biology: a classification. *International Journal of Game Theory* (1986).
- Lotka-Volterra equation and replicator dynamics: new issues in classification. *Biological Cybernetics* (1995).

BOOK

- Together with B. Pötscher: *Game Theoretic Foundations of Evolutionary Stability*, Springer, Berlin (1989).

Costly-signaling games – wide range of applications

- In economics:
  - Spence (1973): education as a costly signal in the job market
  - Miller and Rock (1985): dividend policy as a costly signal in financial markets
  - Milgrom and Roberts (1986): advertising as a costly signal of product quality
- In biology:
  - Zahavi (1975): The Handicap Principle: grounding Darwin's theory of sexual selection through mate choice in natural selection
  - Dawkins and Krebs (1978): emphasize the possibility of "cheating"
  - Grafen (1990): formalizes a strict version of Zahavi's theory
- Sociology and anthropology:
  - Veblen's (1899) theory of Conspicuous Consumption – precursor

– Bliege Bird and Smith (2005): inefficient foraging strategies, gift-giving, communal sharing, rituals, embodied handicaps as costly signals

- The study of language: politeness as a costly signal (van Rooy 2003)

Interest for theorists:

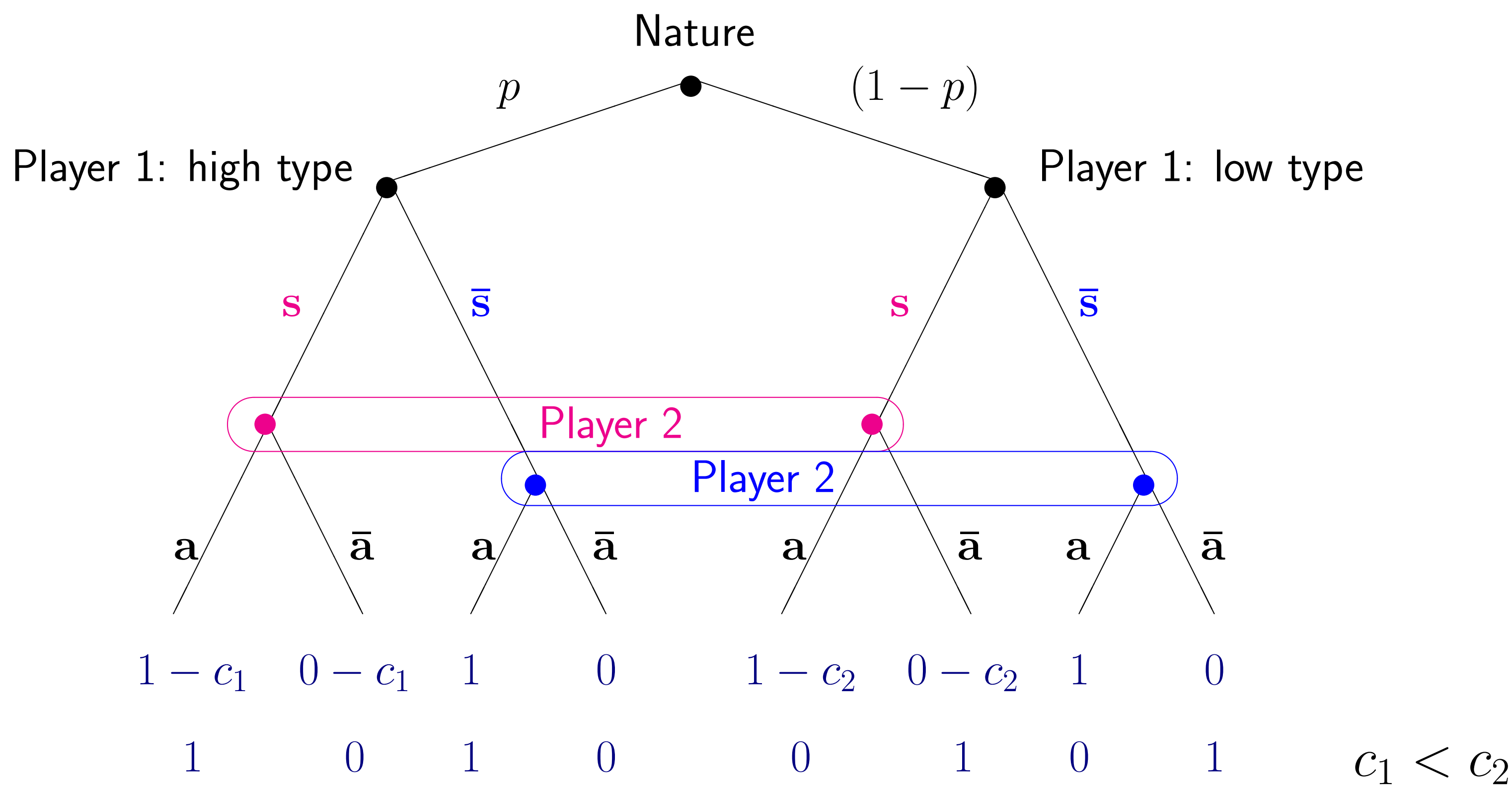
multiplicity of equilibria $\rightarrow$ equilibrium refinement

Banks and Sobel (1987), Cho and Kreps (1987)

Classification of costly-signaling games

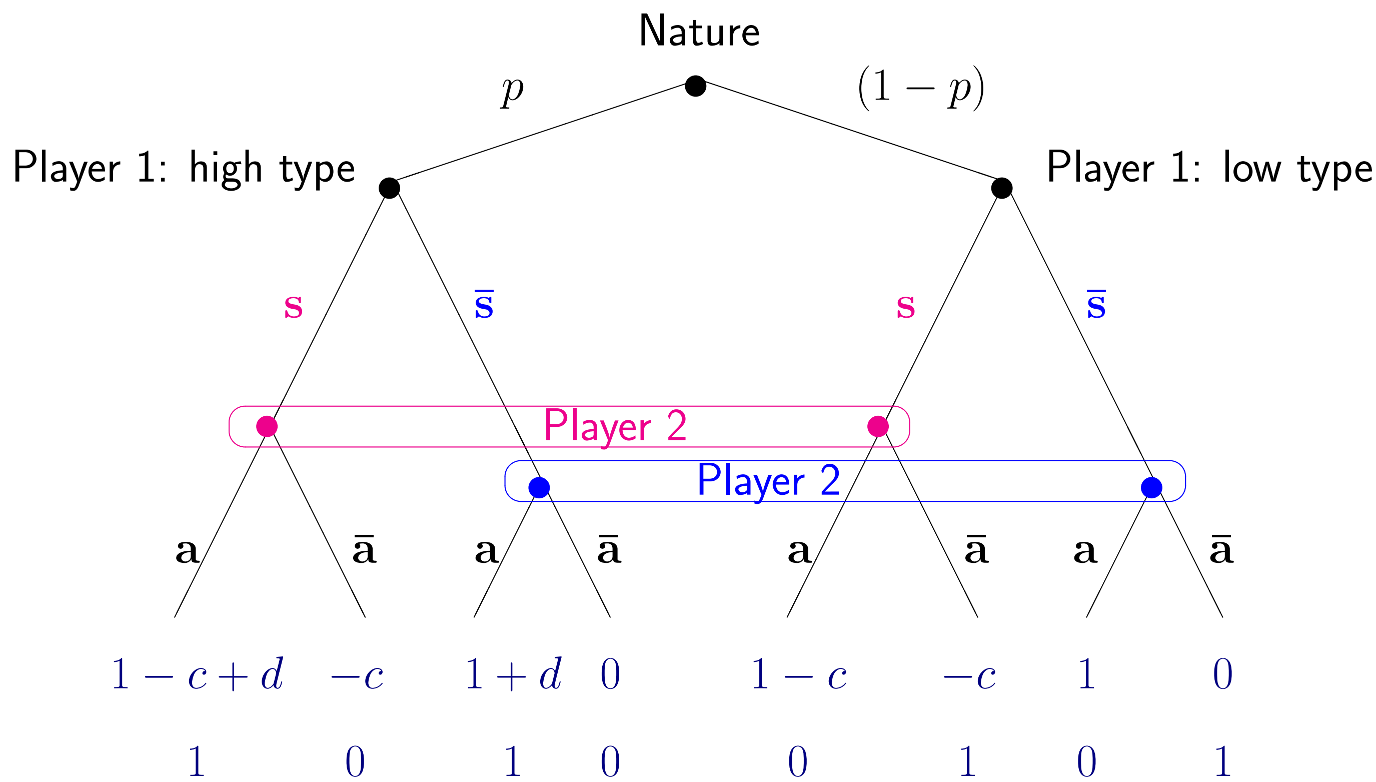
- 2 states of the world, 2 signals, 2 actions
- 5 classes
- Equilibrium refinement:
  - classicial game theory: plausibility-of-beliefs based concepts
  - evolutionary stability based on index theory
- Applications

Class I: different costs in producing the signal (Spence 1973)



	$\mathbf{aa}$	$\mathbf{a\bar{a}}$	$\mathbf{\bar{a}a}$	$\mathbf{\bar{a}\bar{a}}$
$\mathbf{ss}$	$1-pc_1-(1-p)c_2,\ p$	$1-pc_1-(1-p)c_2,\ p$	$-pc_1-(1-p)c_2,\ 1-p$	$-pc_1-(1-p)c_2,\ 1-p$
$\mathbf{s\bar{s}}$	$1-pc_1,\ p$	$p(1-c_1),\ 1$	$-pc_1+(1-p),\ 0$	$-pc_1,\ 1-p$
$\mathbf{\bar{s}s}$	$1-(1-p)c_2,\ p$	$(1-p)(1-c_2),\ 0$	$p-(1-p)c_2,\ 1$	$-(1-p)c_2,\ 1-p$
$\mathbf{\bar{s}\bar{s}}$	$1,\ p$	$0,\ 1-p$	$1,\ p$	$0,\ 1-p$

Class II: uniform costs, differential gains: Handicap Principle



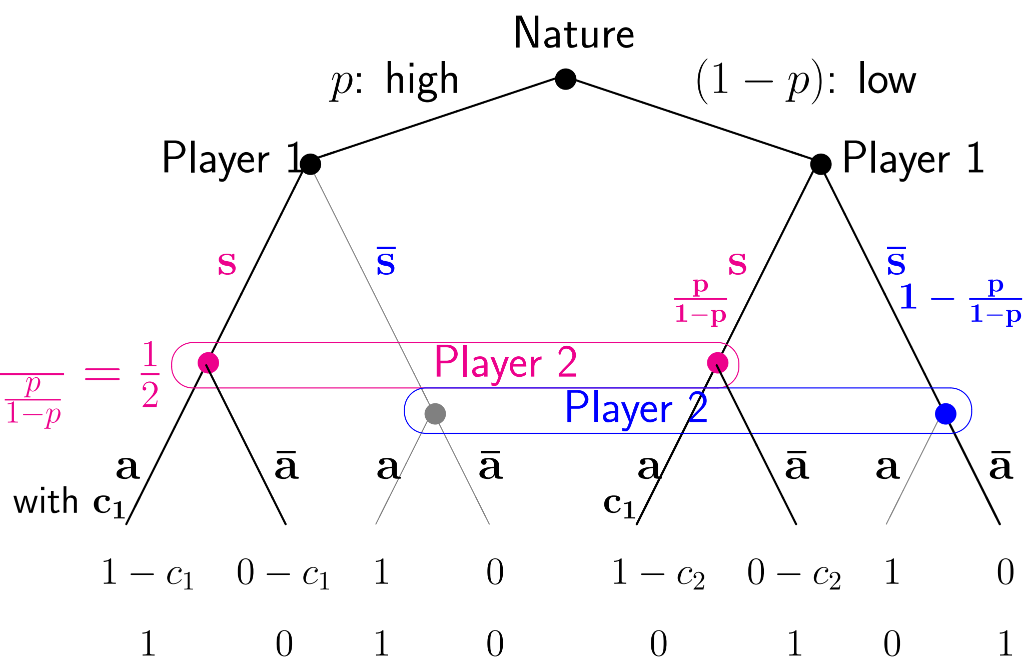
	$\mathbf{aa}$	$\mathbf{a\bar{a}}$	$\mathbf{\bar{a}a}$	$\mathbf{\bar{a}\bar{a}}$
$\mathbf{ss}$	$1-c+pd,\ p$	$1-c+pd,\ p$	$-c,\ 1-p$	$-c,\ 1-p$
$\mathbf{s\bar{s}}$	$1+p(d-c),\ p$	$p(1+d-c),\ 1$	$-pc+(1-p),\ 0$	$-pc,\ 1-p$
$\mathbf{\bar{s}s}$	$1+pd-(1-p)c,\ p$	$(1-p)(1-c),\ 0$	$p(1+d)-(1-p)c,\ 1$	$-(1-p)c,\ 1-p$
$\mathbf{\bar{s}\bar{s}}$	$1+pd,\ p$	$0,\ 1-p$	$1+pd,\ p$	$0,\ 1-p$

Problem: multiplicity of equilibria

- Often focus on fully separating equilibria
  But do not always exist. (If they exist, exist for any prior.)
- There can be also:
  - pooling equilibria: both types use the same signal, which then carries no information. If that signal comes with a cost, waste of social resources! (Depend on the prior!)
  - partially separating equilibria: one signal fully reveals one type, while the other induces a non-trivial Bayesian update of the prior.
    (Depend on the prior!)

$\longrightarrow$ How to select among equilibria? Equilibrium refinement? Rationality-based? Evolutionary criteria?

Class I, $0 < c_1 < c_2 < 1$: $p < \frac{1}{2}$: partially separating equilibrium



Bayes' law:

$$p(h \mid s) = \frac{p}{p + (1-p) \cdot \frac{p}{1-p}} = \frac{1}{2}$$

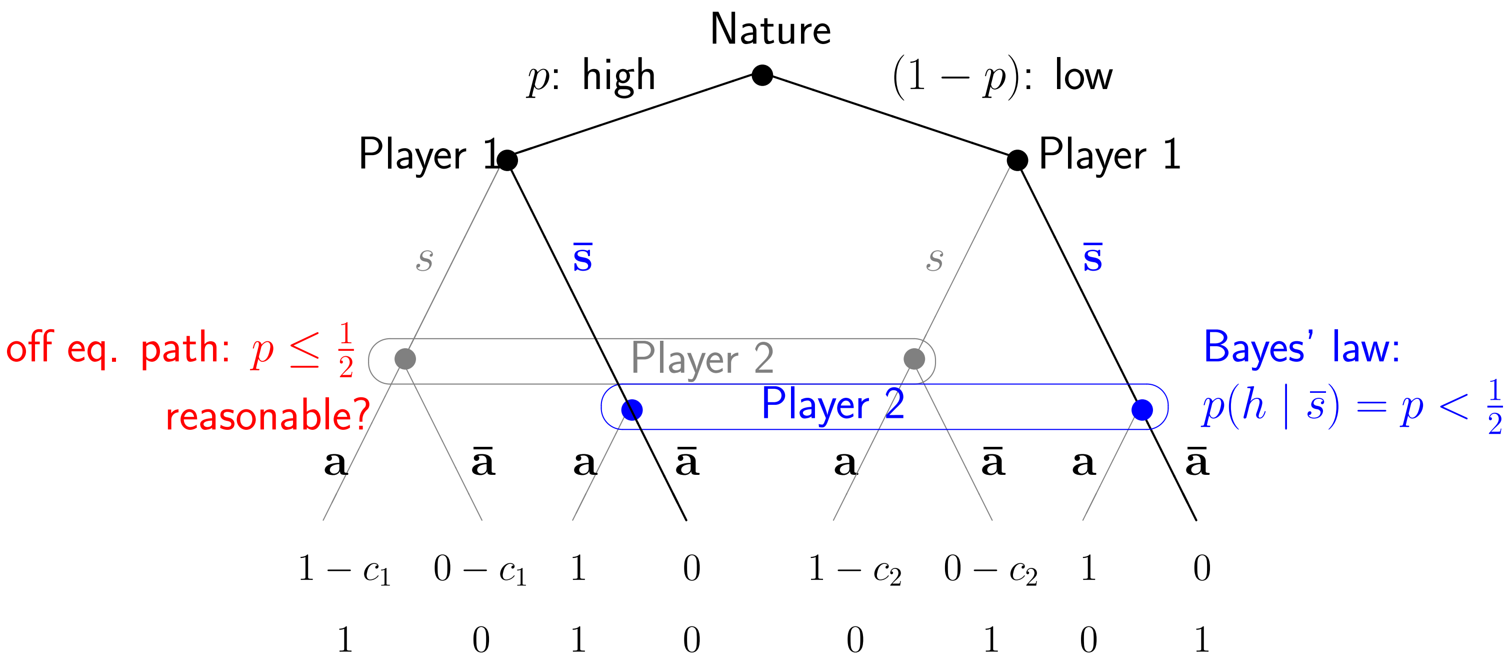
Bayes' law:

$$p(h \mid \bar{s}) = 0$$

		$\mathbf{aa}$	$\mathbf{c_1 \to a\bar{a}}$	$\mathbf{\bar{a}a}$	$\mathbf{1 - c_1 \to \bar{a}\bar{a}}$
$\frac{\mathbf{p}}{\mathbf{1-p}} \to$	$\mathbf{ss}$	$1 - pc_1 - (1-p)c_2,\ p$	$\mathbf{1 - pc_1 - (1-p)c_2},\ p$	$-pc_1 - (1-p)c_2,\ \mathbf{1-p}$	$-pc_1 - (1-p)c_2,\ \mathbf{1-p}$
$\to$	$\mathbf{s\bar{s}}$	$1 - pc_1,\ p$	$p(1-c_1),\ \mathbf{1}$	$-pc_1 + (1-p),\ 0$	$-pc_1,\ 1-p$
	$\mathbf{\bar{s}s}$	$1 - (1-p)c_2,\ p$	$(1-p)(1-c_2),\ 0$	$p - (1-p)c_2,\ \mathbf{1}$	$-(1-p)c_2,\ 1-p$
	$\mathbf{\bar{s}\bar{s}}$	$\mathbf{1},\ p$	$0,\ \mathbf{1-p}$	$\mathbf{1},\ p$	$\mathbf{0},\ \mathbf{1-p}$

- Partially separating: 1 mixes between ss and $s\bar{s}$ with $\frac{p}{1-p}$; 2 between $a\bar{a}$ and $\bar{a}\bar{a}$, with c_2.

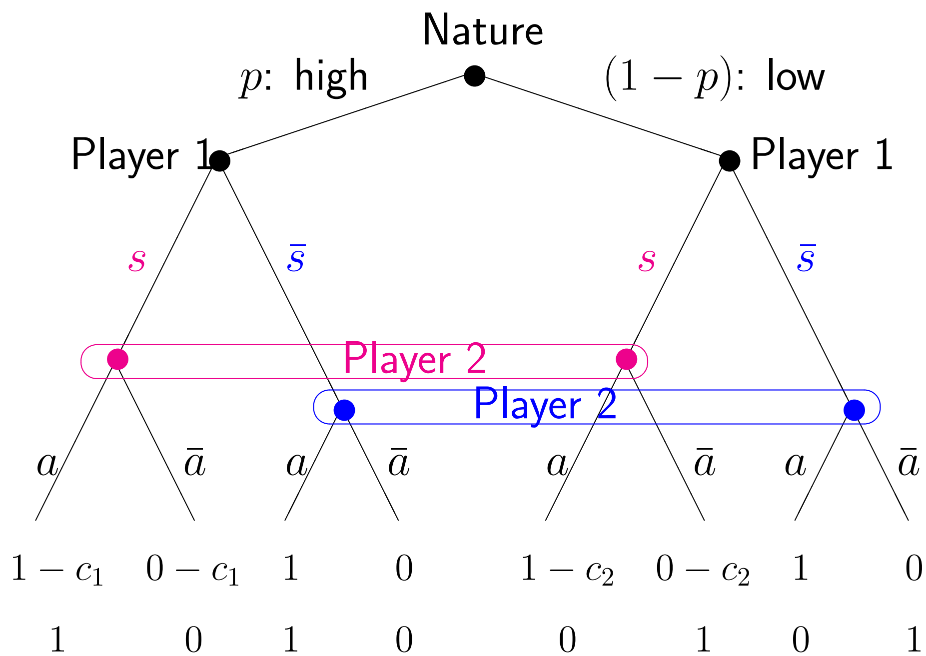
Class I, $0 < c_1 < c_2 < 1$, $p < \frac{1}{2}$: "no-signaling" equilibrium outcome



	$\mathbf{aa}$	with $y \in [0, c_1] \to \mathbf{a\bar{a}}$	$\mathbf{\bar{a}a}$	with $1-y \to \mathbf{\bar{a}\bar{a}}$
$\mathbf{ss}$	$1 - pc_1 - (1-p)c_2,\ p$	$\mathbf{1 - pc_1 - (1-p)c_2},\ p$	$-pc_1 - (1-p)c_2,\ \mathbf{1-p}$	$-pc_1 - (1-p)c_2,\ \mathbf{1-p}$
$\mathbf{s\bar{s}}$	$1 - pc_1,\ p$	$p(1-c_1),\ \mathbf{1}$	$-pc_1 + (1-p),\ 0$	$-pc_1,\ 1-p$
$\mathbf{\bar{s}s}$	$1 - (1-p)c_2,\ p$	$(1-p)(1-c_2),\ 0$	$p - (1-p)c_2,\ \mathbf{1}$	$-(1-p)c_2,\ 1-p$
$\mathbf{\bar{s}\bar{s}}$	$\mathbf{1},\ p$	$0,\ \mathbf{1-p}$	$\mathbf{1},\ p$	$\mathbf{0},\ \mathbf{1-p}$

- No-signaling eq. outcome: 1 takes $\bar{s}\bar{s}$; 2 mix between $a\bar{a}$ and $\bar{a}\bar{a}$ with $y \in [0, c_1]$ on first.

Class I, $0 < c_1 < c_2 < 1$: $p > 1/2 \longrightarrow$ three equilibrium outcomes



	aa	$a\bar{a}$	$\bar{a}a$	$\bar{a}\bar{a}$
ss	$1-pc_1-(1-p)c_2$, $\mathbf{p}$	$\mathbf{1-pc_1-(1-p)c_2}$, $\mathbf{p}$	$-pc_1-(1-p)c_2$, $1-p$	$-pc_1-(1-p)c_2$, $1-p$
$s\bar{s}$	$1-pc_1$, p	$p(1-c_1)$, $\mathbf{1}$	$-pc_1+(1-p)$, 0	$-pc_1$, $1-p$
$\bar{s}s$	$1-(1-p)c_2$, p	$(1-p)(1-c_2)$, 0	$p-(1-p)c_2$, $\mathbf{1}$	$-(1-p)c_2$, $1-p$
$\bar{s}\bar{s}$	$\mathbf{1}$, $\mathbf{p}$	0, $1-p$	$\mathbf{1}$, $\mathbf{p}$	$\mathbf{0}$, $1-p$

- *partially separating equilibrium* in which 1 mixes between $s\bar{s}$ and $\bar{s}\bar{s}$, and 2 between aa and $a\bar{a}$,
- 1 takes ss, and 2 in response to s take a, and
- takes $\bar{s}\bar{s}$, and 2 in response to $\bar{s}$ takes a.

Equilibrium structure, class I, $0 \leq c_1 < c_2 < 1$:

- $p < \frac{1}{2}$:
  - (PS1) *partially separating*:

    $$\mathbf{h} \longrightarrow \mathbf{s} \qquad \mathbf{s} \longrightarrow \mathbf{a} \text{ with } \mathbf{c_2}$$
    $$\mathbf{l} \longrightarrow \mathbf{s} \text{ with } \frac{\mathbf{p}}{\mathbf{1-p}} \qquad \bar{\mathbf{s}} \longrightarrow \bar{\mathbf{a}}$$

  - (P1) *pooling in* $\bar{s}$:

    $$\mathbf{h} \longrightarrow \bar{\mathbf{s}} \qquad \mathbf{s} \longrightarrow \mathbf{a} \ \leq \mathbf{c_1}$$
    $$\mathbf{l} \longrightarrow \bar{\mathbf{s}} \qquad \bar{\mathbf{s}} \longrightarrow \bar{\mathbf{a}}$$

- $p > \frac{1}{2}$:
  - (PS2) *partially separating*:

    $$\mathbf{h} \longrightarrow \bar{\mathbf{s}} \text{ with } \frac{\mathbf{1-p}}{\mathbf{p}} \qquad \mathbf{s} \longrightarrow \mathbf{a}$$
    $$\mathbf{l} \longrightarrow \bar{\mathbf{s}} \qquad \bar{\mathbf{s}} \longrightarrow \mathbf{a} \text{ with } \mathbf{1 - c_1}$$

  - (P2) *pooling in* s:

    $$\mathbf{h} \longrightarrow \mathbf{s} \qquad \mathbf{s} \longrightarrow \mathbf{a}$$
    $$\mathbf{l} \longrightarrow \mathbf{s} \qquad \bar{\mathbf{s}} \longrightarrow \mathbf{a} \text{ with } \ \leq \mathbf{1 - c_2}$$

  - (P3) *pooling in* $\bar{s}$:

    $$\mathbf{h} \longrightarrow \bar{\mathbf{s}} \qquad \mathbf{s} \longrightarrow \mathbf{a} \text{ with any prob.}$$
    $$\mathbf{l} \longrightarrow \bar{\mathbf{s}} \qquad \bar{\mathbf{s}} \longrightarrow \mathbf{a}$$

Equilibrium structure, class I, $0 \leq c_1 \leq 1$, $c_2 > 1$:

- $p < \frac{1}{2}$:
  - (S) *fully separating*: $\mathbf{h} \longrightarrow \mathbf{s}$, $\mathbf{l} \longrightarrow \bar{\mathbf{s}}$; $\mathbf{s} \longrightarrow \mathbf{a}$, $\bar{\mathbf{s}} \longrightarrow \bar{\mathbf{a}}$
  - (P1) *pooling in* $\bar{s}$: $\mathbf{h} \longrightarrow \bar{\mathbf{s}}$, $\mathbf{l} \longrightarrow \bar{\mathbf{s}}$; $\mathbf{s} \longrightarrow \mathbf{a}$ with $\leq \mathbf{c_1}$, $\bar{\mathbf{s}} \longrightarrow \bar{\mathbf{a}}$
- $p > \frac{1}{2}$:
  - (PS2) *partially separating*: $\mathbf{h} \longrightarrow \bar{\mathbf{s}}$ with $\frac{\mathbf{1-p}}{\mathbf{p}}$, $\mathbf{l} \longrightarrow \bar{\mathbf{s}}$; $\mathbf{s} \longrightarrow \mathbf{a}$, $\bar{\mathbf{s}} \longrightarrow \mathbf{a}$ with $\mathbf{1} - \mathbf{c_1}$
  - (S) *fully separating*: $\mathbf{h} \longrightarrow \mathbf{s}$, $\mathbf{l} \longrightarrow \bar{\mathbf{s}}$; $\mathbf{s} \longrightarrow \mathbf{a}$, $\bar{\mathbf{s}} \longrightarrow \bar{\mathbf{a}}$
  - (P3) *pooling in* $\bar{s}$: $\mathbf{h} \longrightarrow \bar{\mathbf{s}}$, $\mathbf{l} \longrightarrow \bar{\mathbf{s}}$; $\mathbf{s} \longrightarrow \mathbf{a}$ with any prob., $\bar{\mathbf{s}} \longrightarrow \mathbf{a}$

Class II: same equilibrium structure as class I:

replace c_1 by $\frac{c}{1+d}$,
and c_2 by c.

Combination of class I and II:

replace c_1 by $\frac{c_1}{1+d}$,
and c_2 by c.

Properties of the equilibrium structure

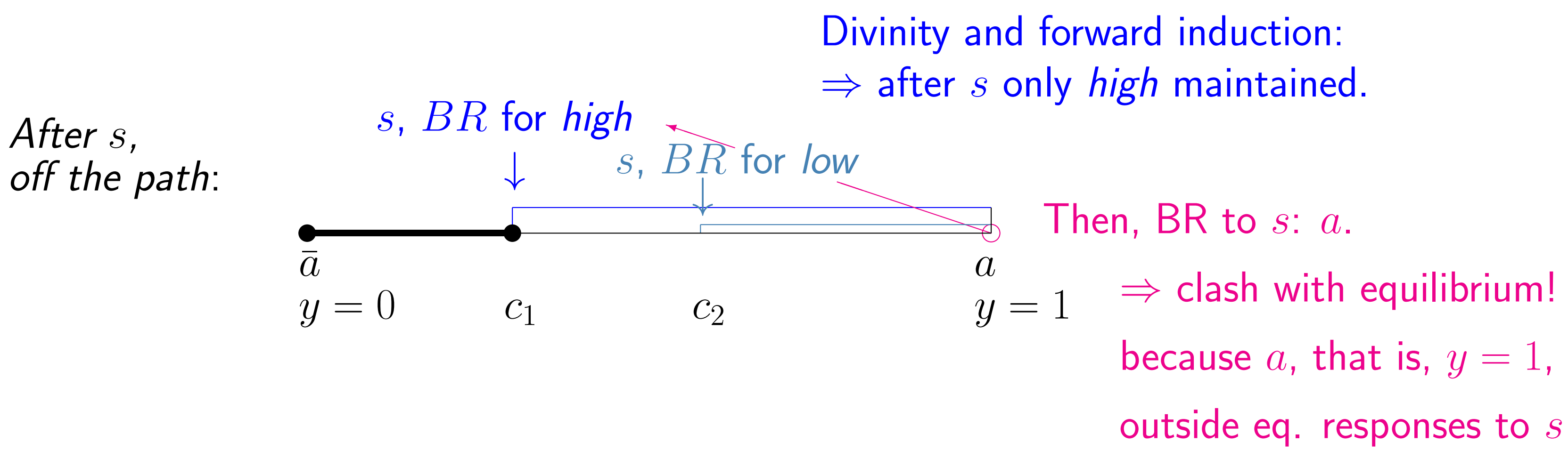
- Which equilibria exist depends on:
  - the cost parameters
  - the prior probability of the state of nature (assumed to be common knowledge)
- Multiple equilibria—even if fully separating equilibria exist!

Equilibrium selection? How to refine the equilibrium notion?

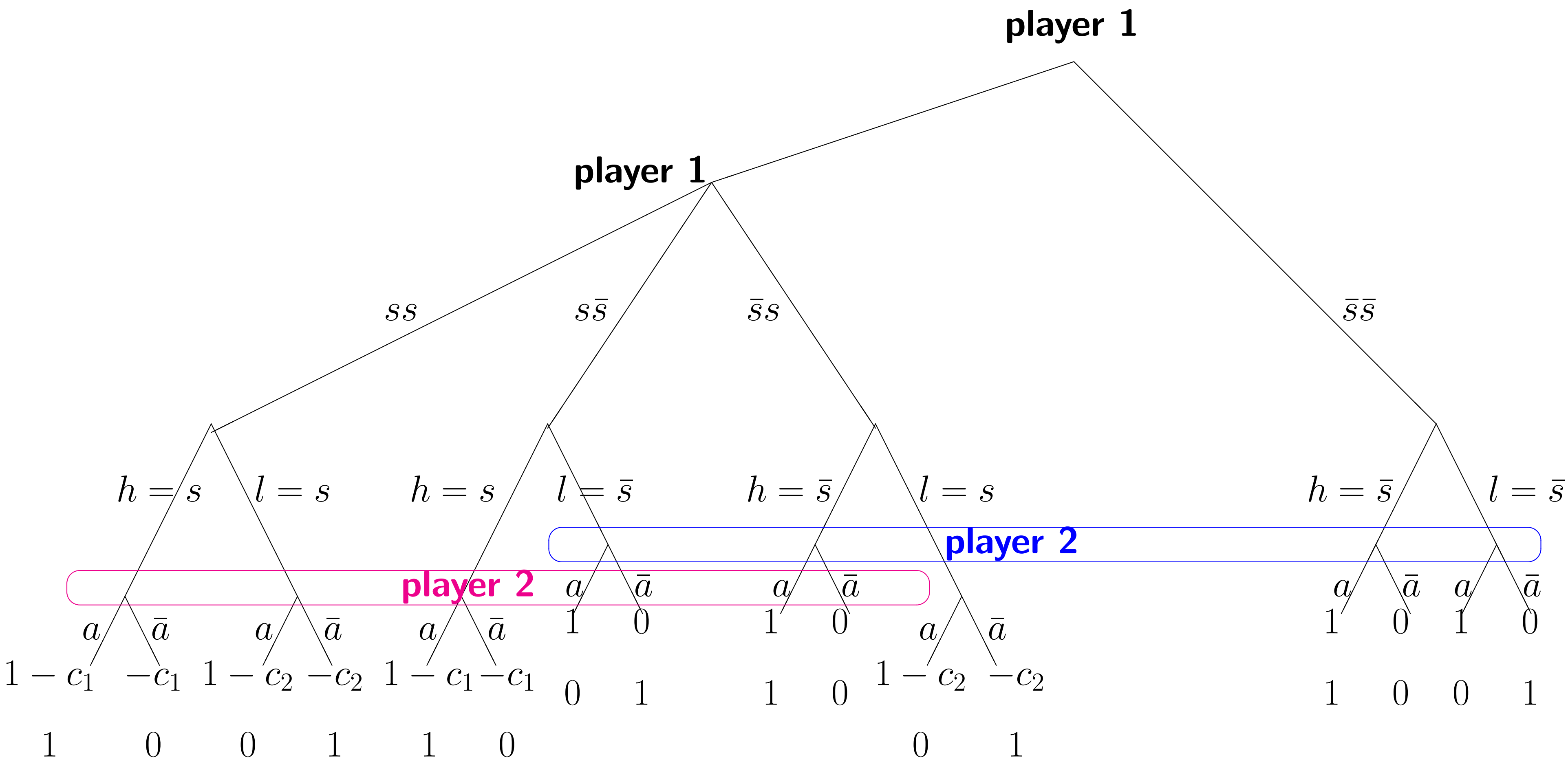
- In classical game theory: $\longrightarrow$ restrictions on beliefs off the equilibrium path
  - Cho and Kreps (1987): "intuitive criterion"
  - Banks and Sobel (1987): "divinity"
  - Govindan and Wilson (2009): "forward induction"

Classical approach: restrictions on beliefs "off the equilibrium path"

No-signaling equilibrium P1 does not satisfy "forward induction" and "divinity":



Forward induction (Govindan and Wilson 2009): foundation in invariance + sequentiality



The index: a rough guide to evolutionary stability

Uses topological properties of the associated fixed point.

If an equilibrium component asymptotically stable under some reasonable dynamics, then index $+1$.

$\Leftrightarrow$

If index $\neq +1$, then equilibrium component cannot be asymptotically stable under any reasonable dynamics.

Index sum: $+1$.

Equilibrium structure: class I, $0 < c_1 < c_2 < 1$

- $p < \frac{1}{2}$:
  - (PS1) *partially separating*:

$h \longrightarrow s$	$s \longrightarrow a$ with c_2	Index $+1$
$l \longrightarrow s$ with $\frac{p}{1-p}$	$\bar{s} \longrightarrow \bar{a}$	forward induction

  - (P1) *pooling in* $\bar{s}$:

$h \longrightarrow \bar{s}$	$s \longrightarrow a$ with $\leq c_1$	Index 0
$l \longrightarrow \bar{s}$	$\bar{s} \longrightarrow \bar{a}$	not forward induction

- $p > \frac{1}{2}$:
  - (PS2) *partially separating*:

$h \longrightarrow \bar{s}$ with $\frac{1-p}{p}$	$s \longrightarrow a$	Index -1
$l \longrightarrow \bar{s}$	$\bar{s} \longrightarrow a$ with $1 - c_1$	forward induction

  - (P2) *pooling in* s:

$h \longrightarrow s$	$s \longrightarrow a$	Index $+1$
$l \longrightarrow s$	$\bar{s} \longrightarrow a$ with $\leq 1 - c_2$	forward induction

  - (P3) *pooling in* $\bar{s}$:

$h \longrightarrow \bar{s}$	$s \longrightarrow a$ with any prob.	Index $+1$
$l \longrightarrow \bar{s}$	$\bar{s} \longrightarrow a$	forward induction

Class	$p < 1/2$	$p = 1/2$	$p > 1/2$
I, $0 \leq c_1 < c_2 < 1$	PS1 / fwd ind / Index $+1$ P1 / not fwd ind / Index 0	PS1-P2 / all fwd ind / Index $+1$ P1-PS2-P3 / not all fwd ind / Index 0	PS2 / fwd ind / Index -1 P2 / fwd ind / Index $+1$ P3 / fwd ind / Index $+1$
I, $0 \leq c_1 < c_2 = 1$	S-PS1 / fwd ind / Index $+1$ P1 / not fwd ind / Index 0	S-PS1-P2 / all fwd ind / Index $+1$ P1-PS2-P3 / not all fwd ind / Index 0	PS2 / fwd ind / Index -1 S-PS1-P2 / fwd ind / Index $+1$ P3 / fwd ind / Index $+1$
I, $0 \leq c_1 \leq 1$, $c_2 > 1$	S / fwd ind / Index $+1$ P1 / not fwd ind / Index 0	S / all fwd ind / Index $+1$ P1-PS2-P3 / not all fwd ind / Index 0	PS2 / fwd ind / Index -1 S / fwd ind / Index $+1$ P3 / fwd ind / Index $+1$
III, $0 < c < 1$	PS1 / fwd ind / Index $+1$ P1 / fwd ind / Index 0	PS1-P2 / all fwd ind / Index $+1$ P1-PS2-P3 / all fwd ind / Index 0	PS2 / fwd ind / Index -1 P2 / fwd ind / Index $+1$ P3 / fwd ind / Index $+1$
III, $c = 1$	S-PS1 / fwd ind / Index $+1$ P1 / fwd ind / Index 0	S-PS1-P2 / all fwd ind / Index $+1$ P1-PS2-P3 / all fwd ind / Index 0	PS2 / fwd ind / Index -1 S-PS1-P2 / fwd ind / Index $+1$ P3 / fwd ind / Index $+1$
III, $c = 1$	S / fwd ind / Index $+1$ P1 / fwd ind / Index 0	S / all fwd ind / Index $+1$ P1-PS2-P3 / all fwd ind / Index 0	PS2 / fwd ind / Index -1 S / fwd ind / Index $+1$ P3 / fwd ind / Index $+1$
IV and V	PS1 / fwd ind / Index $+1$	PS1-P2 / fwd ind / Index $+1$ P1 / not fwd ind / Index 0	P2 / fwd ind / Index $+1$ P1 / not fwd ind / Index 0

Applications in the philosophy and study of language

- Class I: Education as a costly signal $\rightarrow$ language as a carrier of educational signals
- Bourdieu (1982): *Ce que parler veut dire*
  $\rightarrow$ speaking "standard" as a costly signal
- Phenomena explained:
  - Countersignaling (P3)
  - Indirect discrimination (P2)

Sociolinguistics: speaking "standard" as a costly signal (class I)

"Those of us who move from the provinces pay a toll at the city's gate, a toll that is doubled in the years that follow as we try to find a balance between what was so briskly discarded and what was so carefully, hesitantly, slyly put in its place. [...] Did I know, they asked, that my accent and tone, indeed my entire body language, had changed when I met their maid? I was almost a different person. Was I aware that I had, in turn, changed back to the person they had met in Egypt once I was alone with them again ? I asked them, did they not speak in different ways to different people? No, they insisted, they did not. Never! They looked at me as if I was the soul of inauthenticity. And then I realized that those of us who move from the periphery to the center turn our dial to different wavelengths depending on where we are and who else is in the room.

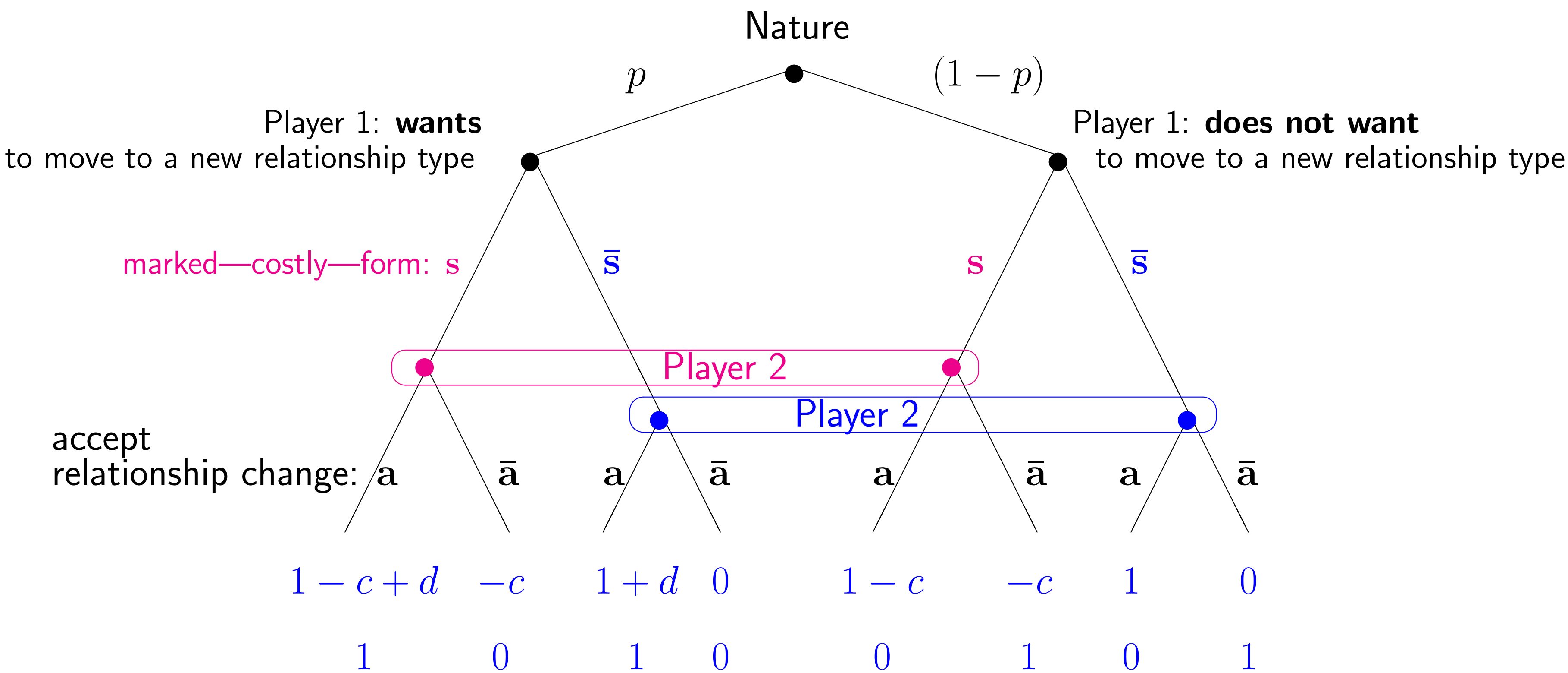
(Colm Tóibín, NYRB, July 13, 2017)

Class II: politeness as a costly signal – relationship negotiation

Costly signal, s	$\longrightarrow$	"marked form"
Absence of the costly signal, $\bar{s}$	$\longrightarrow$	"unmarked form"

Embedded in a relationship negotiation (based on ideas by Fiske 1992, 2004, Pinker at al. 2008, Pinker 2007)

Class II in the context of a relationship negotiation $\longrightarrow$ pragmatics



- Politeness:

  s: Could you pass me the salt? $\bar{s}$: Pass the salt!

  s: Bonjour! Comment allez vous? $\bar{s}$: Bonjour.

Phenomena explained:

- When prior is low:
  - Partially separating equilibrium
    indirect speech $\longrightarrow$ costly signal a means to shape the belief of the other
  - Alternative game tree: talk according to the maxims — flouting of the maxims
- When prior is high:
  - routinely using marked, polite, form (P2) and routinely using unmarked form (P3)—both stable conventions!
    Corresponds to overstatement and understatement!

What constitutes “meaning” in a costly-signaling game?

- “Meaning” arises endogenously in equilibrium
  function of signaling costs and the prior belief

  $\longrightarrow$ what carries meaning is the signal in combination with the prior belief

- Resonates with ideas in the philosophy of language:
  - Davidson (1974): Belief and the basis of meaning.
  - Prior as part of the “context,” “common ground”

Communication: communication about the prior

Prior	Equilibrium	Signal	Posterior	"Meaning"	Reaction
$p < \frac{1}{2}$	part. separating: PS1	s	$p^*_S = \frac{1}{2}$	"increase belief to $\frac{1}{2}$"	a with prob. c_2
		$\bar{s}$	$p^*_{\bar{S}} = 0$	"high type – surely not"	$\bar{a}$
	pooling in $\bar{s}$: P1	$\bar{s}$	$p^*_{\bar{S}} = p < \frac{1}{2}$	"high type not sure enough"	$\bar{a}$
$p > \frac{1}{2}$	part. separating: PS2	s	$p^*_S = 1$	"high type for sure"	a
		$\bar{s}$	$p^*_{\bar{S}} = \frac{1}{2}$	"decrease belief to $\frac{1}{2}$"	a with prob. c_1
	pooling in s: P2	s	$p^*_S = p > \frac{1}{2}$	"high type sure enough"	a
	pooling in $\bar{s}$: P3	$\bar{s}$	$p^*_{\bar{S}} = p > \frac{1}{2}$	"high type sure enough"	a

Happy Birthday, Manuel!